

PHYSICS 439: QUANTUM MECHANICS II
 LECTURE 10: THE HARMONIC OSCILLATOR

Today we will discuss the harmonic oscillator, a fundamental system in quantum mechanics. We will start by reviewing the classical harmonic oscillator and then move on to the quantum case.

The classical harmonic oscillator is described by the Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}kx^2$. The energy levels are continuous, and the motion is periodic with a constant frequency $\omega = \sqrt{k/m}$.

In quantum mechanics, the energy levels are discrete. The ground state energy is $\frac{1}{2}\hbar\omega$, and the energy levels are separated by $\hbar\omega$. The wavefunctions are given by Hermite polynomials multiplied by a Gaussian factor.

The raising and lowering operators, a_+ and a_- , are defined as $a_+ = \frac{1}{\sqrt{2m\hbar\omega}}(p - im\omega x)$ and $a_- = \frac{1}{\sqrt{2m\hbar\omega}}(p + im\omega x)$. They satisfy the commutation relation $[a_-, a_+] = 1$.

The energy eigenstates are $|n\rangle$, where $n = 0, 1, 2, \dots$. The ground state $|0\rangle$ is annihilated by a_- , and the excited states are generated by repeated application of a_+ .

The expectation values of position and momentum in the ground state are zero. The uncertainty in position and momentum is $\Delta x = \sqrt{\frac{\hbar}{2m\omega}}$ and $\Delta p = \sqrt{\frac{\hbar m\omega}{2}}$, respectively.

The harmonic oscillator is a special case of the more general harmonic potential. It is also a good approximation for many physical systems near equilibrium.

Next we will discuss the anharmonic oscillator and the perturbation theory for the harmonic oscillator.